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NONLINEAR SIGNAL PROPAGATION ENHANCED BY NOISE VIA STOCHASTIC RESONANCE

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A model is developed for a nonlinear line of coupled noisy threshold elements. The propagation on the line of various information-carrying signals, periodic, aperiodic or random, is analyzed. Different measures quantifying the efficacy of the propagation are calculated, including signal-to-noise ratio, cross-correlation measures, information-theoretic measures and propagation length. These measures are shown to be improvable by the addition of noise. These results establish a new instance of the nonlinear phenomenon of stochastic resonance under the form of a noise-enhanced propagation applying to a broad variety of signals and noises. The results also contain significance for the propagation of neuronal signals.

1. Introduction

Stochastic resonance [Moss *et al.*, 1994; Gammaitoni *et al.*, 1998] is a nonlinear phenomenon of noise-enhanced signal transmission which has been reported, under different forms, in a broad variety of systems, including, for example, electronic circuits [Fauve & Heslot, 1983; Anishchenko *et al.*, 1992, 1994; Godivier & Chapeau-Blondeau, 1997; Godivier *et al.*, 1997], optical devices [McNamara *et al.*, 1988; Dykman *et al.*, 1995; Jost & Saleh, 1996; Vaudelle *et al.*, 1998], neurons [Bulsara *et al.*, 1991; Douglass *et al.*, 1993; Pantazelou *et al.*, 1995; Chapeau-Blondeau *et al.*, 1996]. Essentially, stochastic resonance has been reported as a noise enhancement of the input–output transmission of a signal applied to an individual nonlinear system. Some studies though, have also considered stochastic resonance in arrays of coupled nonlinear systems [Lindner *et al.*, 1995; Inchiosa & Bulsara, 1995; Jung & Mayer-Kress, 1995], the signal being equally applied to every system of the array and the signal transmission for a system improved by the coupling in the array. Only recently has stochastic resonance been extended to coupled systems where the signal is applied only to one system, and where the effect is interpreted as a noise enhancement of the propagation of the signal among the coupled systems [Löcher *et al.*, 1998; Lindner *et al.*, 1998; Zhang *et al.*, 1998]. Such conditions may be relevant to wave propagation among nonlinear cells like neurons or over excitable or nonlinear media like those supporting solitons. The few studies that have appeared on this matter [Löcher *et al.*, 1998; Lindner *et al.*, 1998; Zhang *et al.*, 1998] usually resort to numerical simulations or experiments, because the coupled nonlinear systems used to exhibit a noise-enhanced propagation form a setting which is complicated enough to defeat an exact theoretical analysis.

Very recently though, a nonlinear line of simple threshold elements has been shown capable of a noise-enhanced propagation via an exact theoretical treatment backed up by an experiment [Chapeau-Blondeau, 1999]. The noise-enhanced propagation was demonstrated, both theoretically and experimentally, for a sinusoidal signal, with quantification by the standard signal-to-noise ratio of periodic stochastic resonance. Here, we shall consider a similar type of nonlinear line as in [Chapeau-Blondeau, 1999], yet with a slight modification to allow for a neuronal interpretation. We shall extend the model to demonstrate noise-enhanced propagation of both periodic and aperiodic signals, and also of deterministic and random information-carrying signals. We shall extend the quantification measures to incorporate, in addition to the standard signal-to-noise ratio, cross-correlation measures and information-theoretic measures, and also define a propagation length that we show increasable via the addition of noise for the propagation of various information-carrying signals.

2. The Nonlinear Propagation Line

We consider the propagation line formed, as in Fig. 1, by the cascade of nonlinear cells consisting of two-state threshold elements. The input end of the line is fed by an information-carrying signal $s(t)$. The cell $n \geq 1$ receives at its input the sum $y_{n-1}(t)+\eta_n(t) = x_n(t)$ where $\eta_n(t)$ is the local noise on cell n, and $y_{n-1}(t)$ is the output signal from cell $n-1$ except for the first cell $n = 1$ for which the input $y_0(t) \equiv s(t)$. We consider that the local noises are independent from cell to cell, and each $\eta_n(t)$ is white and stationary although not necessarily Gaussian.

Each cell $n \geq 1$ produces a two-state output $y_n(t)$ according to

$$\text{If } x_n(t) > \theta \quad \text{then } \; y_n(t) = 1 \qquad \text{else } \; y_n(t) = 0\,, \tag{1}$$

with the threshold θ which, for simplicity, is assumed to be the same for all cells. Such nonlinear cells can be seen as mimicking, in certain conditions, the intrinsic nonlinearity of a neuron which generates a nonzero output activity when its input exceeds a given threshold.

At cell n, the output $y_n(t)$ results as a random signal bearing some dependency with the information-carrying input $s(t)$ propagating down the noisy line. We shall introduce quantitative measures, according to the nature of $s(t)$, to express this dependency. We shall then show that regimes exist in the propagation where these measures can be improved by the addition of noise, thus establishing an instance of stochastic resonance under the form of a noise-enhanced propagation.

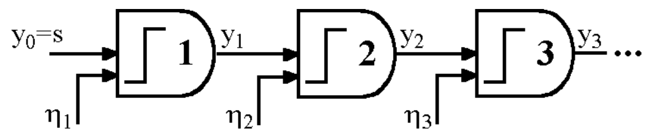

Fig. 1. Nonlinear line cascading two-state threshold elements.

3. Periodic Signal

We first consider the case where the information-carrying $s(t)$ is a T_s-periodic signal. In this case, an appropriate measure to quantify a noise-enhanced propagation is the standard signal-to-noise ratio (SNR) defined in the frequency domain for periodic stochastic resonance [Gammaitoni *et al.*, 1998]. With a T_s-periodic input $s(t)$, the output $y_n(t)$ at cell n is a cyclostationary random signal with period T_s [Chapeau-Blondeau & Godivier, 1997]. As a result, the power spectral density of $y_n(t)$ is formed [Moss *et al.*, 1994; Chapeau-Blondeau & Godivier, 1997] by spectral lines at integer multiples of $1/T_s$ emerging out of a broadband continuous noise background. On the output of cell n, a standard signal-to-noise ratio (SNR) $\mathcal{R}_n$ is defined [Moss *et al.*, 1994; Chapeau-Blondeau & Godivier, 1997] as the power contained in the coherent spectral line at $1/T_s$ divided by the power contained in the noise background in a small frequency band ΔB around $1/T_s$. According to the theory of [Chapeau-Blondeau & Godivier, 1997], this SNR can be expressed as

$$\mathcal{R}_n = \frac{\left| \left\langle \mathrm{E}[y_n(t)] \, \exp\left(\dfrac{-\,i2\pi t}{T_s} \right) \right\rangle \right|^2}{\langle \mathrm{var}[y_n(t)] \rangle \Delta t \Delta B}\,, \tag{2}$$

with the time average defined as

$$\langle \cdots \rangle = \frac{1}{T_s} \int_0^{T_s} \ldots \, dt\,. \tag{3}$$

Also Δt is the time resolution of the measurement (i.e. the signal sampling period in a discrete time implementation).

For the two-state cell of Eq. (1), we have for the output expectation

$$\mathrm{E}[y_n(t)] = \Pr\{y_n(t) = 1\} = 1 - \Pr\{y_n(t) = 0\}\,, \tag{4}$$

and for the output variance

$$\mathrm{var}[y_n(t)] = \Pr\{y_n(t) = 0\}[1 - \Pr\{y_n(t) = 0\}] \quad (5)$$

since $\mathrm{E}[y_n^2(t)] = \mathrm{E}[y_n(t)]$.

To compute the SNR $\mathcal{R}_n$ for cell n with Eqs. (2)–(5), we need to relate both $\mathrm{E}[y_n(t)]$ and $\mathrm{var}[y_n(t)]$ to the coherent input $s(t)$ to the line and to the properties of the noise sources down the line.

The input $x_n(t)$ to cell n is a random signal whose cumulative distribution function $F_{x_n}(u) = \Pr\{x_n(t) \le u\}$ verifies

$$F_{x_n}(u) = \Pr\{y_{n-1} = 0\}F_{\eta_n}(u) + \Pr\{y_{n-1} = 1\}F_{\eta_n}(u-1)\,, \quad (6)$$

where $F_{\eta_n}(u)$ is the cumulative distribution of the noise $\eta_n(t)$. The two-state ouput $y_n(t)$ thus occurs with probabilities $\Pr\{y_n(t) = 0\} = \Pr\{x_n(t) \le \theta\} = F_{x_n}(\theta)$ and $\Pr\{y_n(t) = 1\} = 1 - F_{x_n}(\theta)$. Thanks to Eq. (6), we can write

$$\Pr\{y_n = 0\} = F_{x_n}(\theta) = F_{x_{n-1}}(\theta)Q_n(\theta) + F_{\eta_n}(\theta - 1)\,, \quad (7)$$

with $Q_n(\theta) = F_{\eta_n}(\theta) - F_{\eta_n}(\theta - 1)$. Applying a chain rule, we can obtain $\Pr\{y_n(t) = 0\}$ as a function of $F_{x_1}(\theta) = F_{\eta_1}[\theta - s(t)]$ and of the $F_{\eta_j}(\theta)$'s and $F_{\eta_j}(\theta - 1)$'s for $j = 2$ to n. For the sake of simplicity, we choose to write this expression in the case where the noises $\eta_j(t)$, although independent, share the same cumulative distribution $F_\eta(u)$, yielding

$$\Pr\{y_n = 0\} = F_\eta[\theta - s(t)]Q^{n-1}(\theta) + F_\eta(\theta - 1)\,\frac{1 - Q^{n-1}(\theta)}{1 - Q(\theta)}\,, \quad (8)$$

with $Q(\theta) = F_\eta(\theta) - F_\eta(\theta - 1)$.

Equation (8) together with Eqs. (2)–(5) then provide an explicit expression for the SNR $\mathcal{R}_n$ at cell n, for a T_s-periodic input $s(t)$ with arbitrary waveform feeding the line corrupted by noise with arbitrary distribution $F_\eta(u)$. A study of the evolution of $\mathcal{R}_n$ with the noise properties can then be performed, to investigate the conditions for a stochastic resonance in the propagation of the periodic signal $s(t)$.

For illustration we consider $s(t)$ to be the T_s-periodic pulse train defined by $s(t) = 1$ for $t \in [0,\, 0.2T_s]$ and $s(t) = 0$ elsewhere in the period T_s. Such an $s(t)$ can provide a schematized picture of a periodic train of action potentials propagating down a chain of noisy neurons responding to Eq. (1). The local noises η_n would represent the membrane potential random fluctuations at the level of each neuron, originating for instance in random gating of ion channels of the membrane. A train of solitons could be another interpretation for $s(t)$.

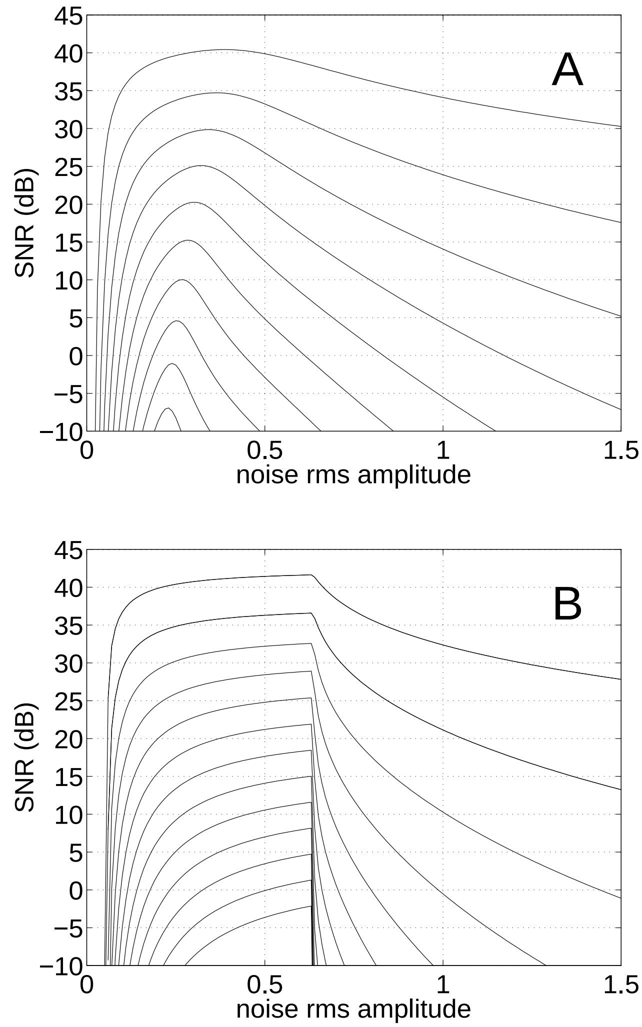

Fig. 2. Output SNR $\mathcal{R}_n$ from Eqs. (2)–(5) and (8) at various cells n, as a function of the rms amplitude of the noise over the line, when the input signal $s(t)$ is a periodic pulse train. Panel A: with zero-mean Gaussian noise, for $n = 1$ to 10 from the upper to the lowest curve. Panel B: with zero-mean uniform noise, for $n = 1$ to 13 from the upper to the lowest curve.

The measure of the propagation efficacy provided by the SNR $\mathcal{R}_n$ has been evaluated from Eqs. (2)–(5) and (8) for different cells n down the line, and with the choice $\Delta t \Delta B = 10^{-5}$. Figure 2 shows this SNR $\mathcal{R}_n$ for different cells n, and for different noise distributions, with cells of threshold $\theta = 1.1$. With such a value of the threshold, the periodic input $s(t)$ and every ouput y_n are below the

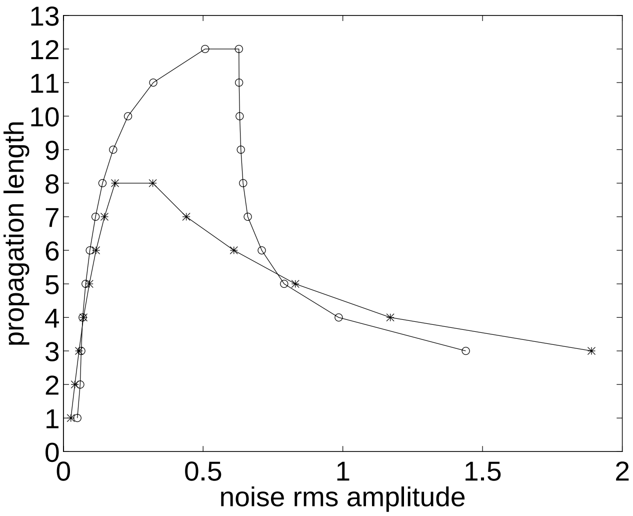

Fig. 3. Propagation length deduced from the SNR for the periodic signal of Fig. 2, as a function of the rms amplitude of the noise over the line. (∗) with zero-mean Gaussian noise. (○) with zero-mean uniform noise.

threshold θ. As a consequence, no propagation can take place in the absence of noise. This translates into an SNR $\mathcal{R}_n$ for every cell n which goes to zero at zero noise, as indicated in Fig. 2. When noise is added over the line, a cooperative effect takes place at each cell, through which the local noise assists the signal in overcoming the threshold. This allows the propagation of the coherent signal down the line, with an SNR $\mathcal{R}_n$ for every cell n which can be maximized by an optimal nonzero noise level, as visible in Fig. 2. If ever the noise were set to zero at any cell n then the propagation would be blocked at this cell. This is an instance of a stochastic resonance effect, under the form of a noise-assisted propagation of a periodic signal.

Another characterization of the noise-assisted propagation that can be deduced from SNR curves like those of Fig. 2, is the evaluation of a propagation length defined as the index n of the remotest cell that can be reached before the SNR $\mathcal{R}_n$ drops below an arbitrary level, say 0 dB. Such a propagation length is shown in Fig. 3, when the input signal $s(t)$ is the periodic pulse train of Fig. 2, as a function of the level of noise over the line. Again, we observe in Fig. 3 that the propagation length goes to zero at zero noise, and that there exists an optimal nonzero noise level where the propagation length is maximized, thus providing another characterization of a stochastic resonance in the propagation of a periodic signal.

4. Aperiodic Signal

4.1. *Deterministic aperiodic signal*

We now consider the case where the information-carrying signal $s(t)$ feeding the line in Fig. 1, is a deterministic aperiodic signal defined over the duration T_s. In such a case, as candidate measures to quantify a noise-enhanced propagation, we can take cross-correlation measures similar to those of [Collins *et al.*, 1995] for aperiodic stochastic resonance. We choose here to use the time-averaged normalized cross-covariance, between the input $s(t)$ and the output $y_n(t)$ of cell n. We introduce the signals centered around their time-averaged statistical expectation,

$$\tilde{s}(t) = s(t) - \langle s(t) \rangle \tag{9}$$

and

$$\tilde{y}_n(t) = y_n(t) - \langle \mathrm{E}[y_n(t)] \rangle \,, \tag{10}$$

with the time average again defined by Eq. (3). The time-averaged normalized cross-covariance is

$$C_{sy_n} = \frac{\langle \mathrm{E}[\tilde{s}(t)\tilde{y}_n(t)] \rangle}{\sqrt{\langle \mathrm{E}[\tilde{s}^2(t)] \rangle \langle \mathrm{E}[\tilde{y}_n^2(t)] \rangle}} \,, \tag{11}$$

or equivalently, since $s(t)$ is deterministic,

$$C_{sy_n} = \frac{\langle \tilde{s}(t)\, \mathrm{E}[\tilde{y}_n(t)] \rangle}{\sqrt{\langle \tilde{s}^2(t) \rangle \langle \mathrm{E}[\tilde{y}_n^2(t)] \rangle}} \,. \tag{12}$$

The expectations $\mathrm{E}[y_n(t)] = \mathrm{E}[y_n^2(t)]$ are again expressed by Eq. (4). And in the case where the local noises $\eta_j(t)$ share the same distribution, Eq. (8) applies to provide these expectations as functions of both $s(t)$ and the common noise distribution $F_\eta(u)$. For a given $s(t)$ the time averages involved in Eq. (12) can then be explicitly realized, possibly through numerical integration, to yield the cross-covariance C_{sy_n} as a function of the noise properties conveyed by $F_\eta(u)$. A study of the evolution of C_{sy_n} with the noise properties can then be performed, to investigate the conditions for a stochastic resonance in the propagation of the deterministic aperiodic signal $s(t)$.

For illustration we consider $s(t)$ to be the aperiodic pulse train of duration T_s depicted in Fig. 4. As before, our treatment applies to an arbitrary waveform for $s(t)$, but the choice of a pulse train for $s(t)$

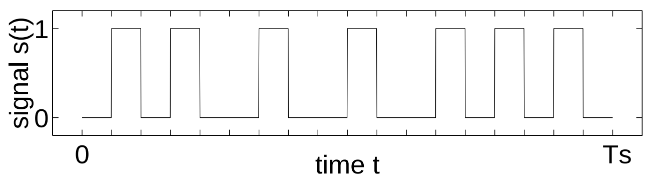

Fig. 4. Deterministic aperiodic signal $s(t)$ of duration T_s experiencing a noise-enhanced propagation.

maintains the possibility of carrying on the neuronal interpretation, with our illustration here picturing the propagation of an aperiodic train of action potentials over a chain of noisy neurons responding to Eq. (1).

The measure of the propagation efficacy provided by the cross-covariance C_{sy_n} from Eq. (12) has been evaluated at different cells n down the line. Figure 5 shows the cross-covariance C_{sy_n} at different cells n, and for different noise distributions, with cells of threshold $\theta = 1.1$. Again, with such a value of the threshold, the input $s(t)$ and every ouput y_n are below the threshold θ. No propagation can take place at zero noise, and the propagation is maximized at each cell by a nonzero noise level, as expressed by the resonant evolutions of the cross-covariance C_{sy_n} of Fig. 5. Again, if the noise is set to zero at any cell, then the propagation of the signal stops at this cell. This is a stochastic resonance effect as a noise-assisted propagation of an aperiodic signal.

Another characterization of the noise-assisted propagation that can be deduced from cross-covariance curves like those of Fig. 5, is the evaluation of a propagation length defined as the index n of the remotest cell that can be reached before the cross-covariance C_{sy_n} drops below an arbitrary level, say -20 dB. Such a propagation length is shown in Fig. 6, when the input signal $s(t)$ is the aperiodic pulse train of Figs. 4 and 5, as a function of the level of the noise over the line. Again, we observe in Fig. 6 that the propagation length goes to zero at zero noise, and that an optimal nonzero noise level maximizes the propagation length, thus characterizing a stochastic resonance in the propagation of a deterministic aperiodic signal.

4.2. *Random aperiodic signal*

We now consider the case where the information-carrying signal $s(t)$ feeding the line of Fig. 1, is a stationary random signal. In such a case, a possibility to quantify a noise-enhanced propagation

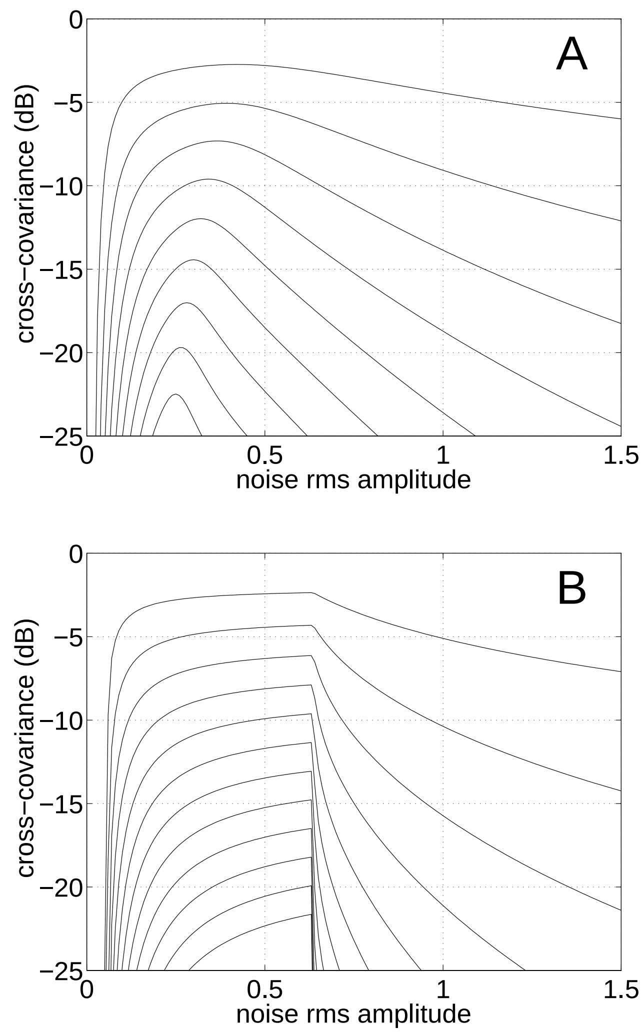

Fig. 5. Normalized cross-covariance C_{sy_n} from Eq. (12) at various cells n, as a function of the rms amplitude of the noise over the line, when the input signal $s(t)$ is the aperiodic pulse train in Fig. 4. Panel A: with zero-mean Gaussian noise, for $n = 1$ to 9 from the upper to the lowest curve. Panel B: with zero-mean uniform noise, for $n = 1$ to 12 from the upper to the lowest curve.

is again to use the normalized cross-covariance expressed by Eq. (12), provided the time average defined in Eq. (3) is replaced by the statistical average

$$\langle \cdots \rangle = \int_s \cdots f_s(s) ds \,, \tag{13}$$

where $f_s(s)$ is the probability density function of the random signal $s(t)$.

Another possibility to quantify a noise-enhanced propagation of a random signal $s(t)$ down the line of Fig. 1, is to rely on information-theoretic measures [Neiman *et al.*, 1996; Heneghan *et al.*, 1996; Bulsara & Zador, 1996; Chapeau-Blondeau, 1997; Godivier & Chapeau-Blondeau, 1998] to

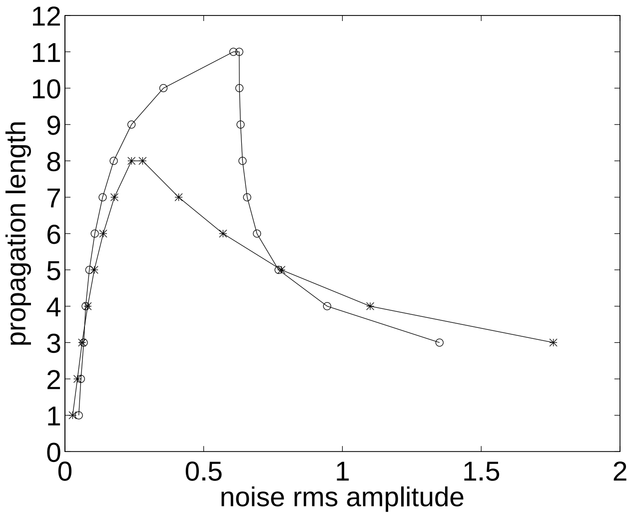

Fig. 6. Propagation length deduced from the cross-covariance for the deterministic aperiodic signal of Figs. 4 and 5, as a function of the rms amplitude of the noise over the line. ($*$) with zero-mean Gaussian noise. ($\circ$) with zero-mean uniform noise.

evaluate a mutual information between $s(t)$ and the output $y_n(t)$ at cell n. Although such a characterization can be realized for general conditions concerning the input signal $s(t)$, we shall consider here special conditions where the characterization can be developed analytically up to the computation of the information capacity of the nonlinear line.

We consider the case where the nonlinear line in Fig. 1 is operated as a memoryless binary information channel. The signals $s(t)$ and every $y_n(t)$ are observed or sampled at discrete times t_k, the distribution of which need not be further specified. The random signal $s(t)$ is assumed to be a white noise, and thus the values $s(t_k)$ for different times t_k are independent. Further, we suppose that the signal s at the sampling times t_k assumes values restricted to 1 or 0, respectively with probabilities $\Pr\{s=1\}=p_1$ and $\Pr\{s=0\}=1-p_1$. At the output of a cell n governed by Eq. (1), the signal y_n at the sampling times also assumes values 0 or 1.

The above conditions can again receive a neuronal interpretation. The sampling times can be taken at regular intervals $t_k = kT$, where T can be interpreted as a neuron refractory period setting the fastest repetition period at which action potentials can be fired by a neuron. The values 0/1 assumed by the signals model the absence/presence of an action potential at the corresponding sampling time. The line model can then be viewed as describing the propagation of an information-carrying train s of action potentials over a chain of noisy neurons.

Between the input s and the output y_n at cell n, the line also represents a memoryless binary information channel [Cover & Thomas, 1991]. The input–output transmission probabilities of this channel can be expressed by means of Eq. (8), again when the local noises share the same distribution. When $s=0$ in Eq. (8), we get the transmission probability

$$\begin{aligned} p_{00} &= \Pr\{y_n=0|s=0\} \\ &= F_\eta(\theta)Q^{n-1}(\theta) \\ &\quad + F_\eta(\theta-1)\,\frac{1-Q^{n-1}(\theta)}{1-Q(\theta)}\,, \end{aligned} \tag{14}$$

and we deduce

$$p_{10} = \Pr\{y_n=1|s=0\} = 1-p_{00}\,. \tag{15}$$

When $s=1$ in Eq. (8), we get the transmission probability

$$\begin{aligned} p_{01} &= \Pr\{y_n=0|s=1\} \\ &= F_\eta(\theta-1)Q^{n-1}(\theta) \\ &\quad + F_\eta(\theta-1)\,\frac{1-Q^{n-1}(\theta)}{1-Q(\theta)}\,, \end{aligned} \tag{16}$$

and we deduce

$$p_{11} = \Pr\{y_n=1|s=1\} = 1-p_{01}\,. \tag{17}$$

Once the input–output transmission probabilities are known, the input–output mutual information $I(s;y_n)$ of the channel can be computed from the entropies as [Cover & Thomas, 1991; Chapeau-Blondeau, 1997]

$$I(s;y_n) = H(y_n) - H(y_n|s)\,. \tag{18}$$

The output entropy at cell n can be expressed as

$$\begin{aligned} H(y_n) &= h[p_1p_{11} + (1-p_1)p_{10}] \\ &\quad + h[p_1p_{01} + (1-p_1)p_{00}]\,, \end{aligned} \tag{19}$$

with the function $h(u) = -u\log_2(u)$, and the input–output conditional entropy as

$$\begin{aligned} H(y_n|s) &= p_1[h(p_{11}) + h(p_{01})] \\ &\quad + (1-p_1)[h(p_{00}) + h(p_{10})]\,. \end{aligned} \tag{20}$$

Equations (18)–(20) provide an explicit expression for the mutual information $I(s;y_n)$ as a function of the input probability p_1. The derivative of

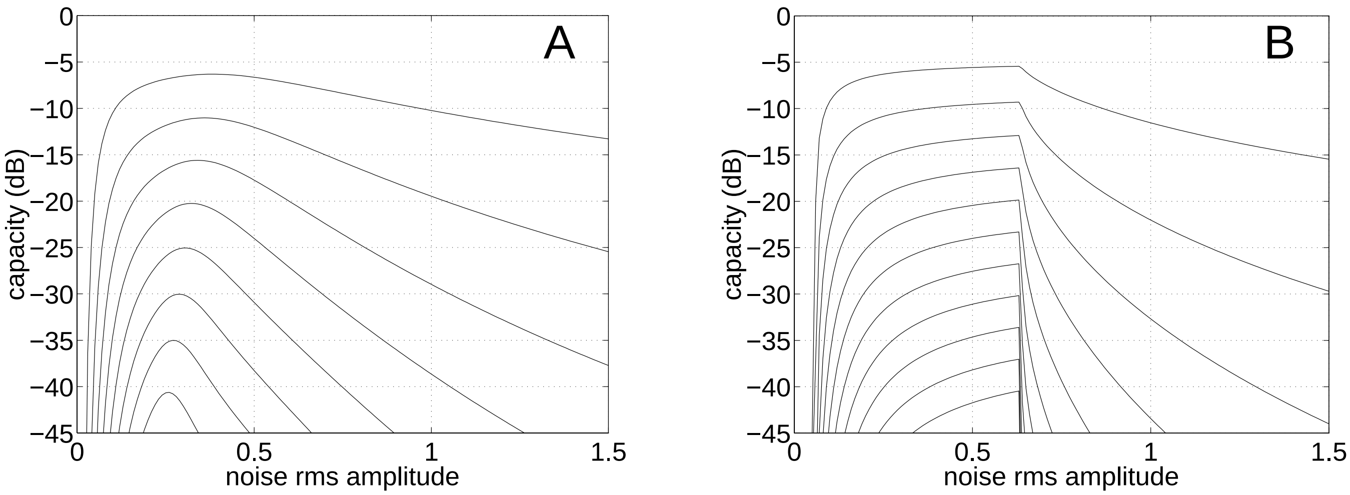

Fig. 7. Information capacity C_n from Eqs. (18)–(22) at various cells n, as a function of the rms amplitude of the noise over the line, when the input signal $s(t)$ is a random pulse train. Panel A: with zero-mean Gaussian noise, for $n = 1$ to 8 from the upper to the lowest curve. Panel B: with zero-mean uniform noise, for $n = 1$ to 11 from the upper to the lowest curve.

$I(s;\, y_n)$ relative to p_1 can be computed, to yield the value p_1^* of p_1 that maximizes $I(s;\, y_n)$ and achieves the channel capacity C_n; this value comes out as

$$p_1^* = \frac{ap_{00} - 1}{a(p_{00} - p_{01})}\,, \tag{21}$$

with

$$a = 1 + \exp\left[\ln(2)\frac{h(p_{00}) + h(p_{10}) - h(p_{11}) - h(p_{01})}{p_{00} - p_{01}}\right]. \tag{22}$$

Expression (21) used in Eqs. (19) and (20) results in an explicit expression for the channel capacity C_n as the maximum $I(s;\, y_n)$ which follows in Eq. (18). A study of the evolution of C_n with the noise properties can then be performed, to investigate the conditions for a stochastic resonance in the propagation of the random signal $s(t)$.

The measure of the propagation efficacy provided by the capacity C_n has been evaluated for different cells n down the line. Figure 7 shows this capacity C_n at different cells n, and for different noise distributions, with cells of threshold $\theta = 1.1$. Again, with such a value of the threshold, the input s and every ouput y_n are below the threshold θ. A nonzero noise is strictly necessary at each cell to have information propagating through the line. The flow of information is maximized at an optimal nonzero noise level, as expressed by the resonant evolutions of the capacity C_n in Fig. 7. If the noise is set to zero at any cell, then the flow of information stops at this cell. This is a stochastic resonance effect as a noise-assisted propagation of information down the line.

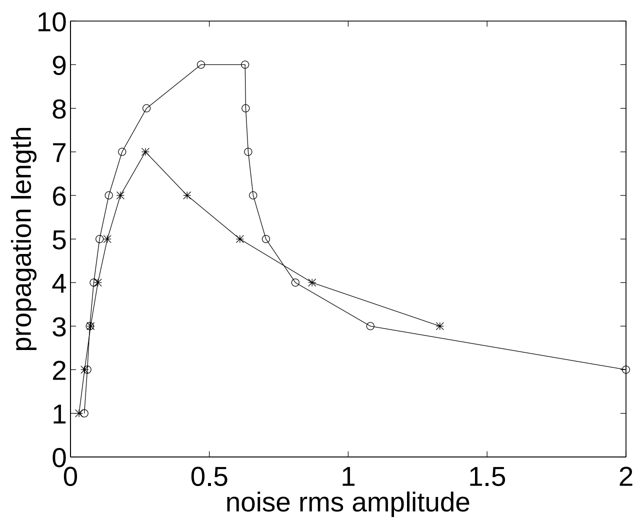

Fig. 8. Propagation length deduced from the information capacity for the random aperiodic signal of Fig. 7, as a function of the rms amplitude of the noise over the line. (∗) with zero-mean Gaussian noise. (○) with zero-mean uniform noise.

Another characterization of the noise-assisted propagation that can be deduced from information-capacity curves like those in Fig. 7, is the evaluation of a propagation length defined as the index n of the remotest cell that can be reached before the capacity C_n drops below an arbitrary level, say -35 dB. Such a propagation length is shown in Fig. 8 for the conditions of Fig. 7, as a function of the level of the noise over the line. Again, we observe in Fig. 8 the propagation length which goes to zero at zero noise and the existence of an optimal nonzero noise level where the propagation length is maximized, thus characterizing a stochastic resonance in the propagation of information.

5. Conclusion

The present model establishes a new instance of the nonlinear phenomenon of stochastic resonance under the form of a noise-enhanced propagation, over a line of threshold elements, which applies to a broad variety of signals and noises. Here, we have considered the information-carrying signal $s(t)$ under the form of pulse trains (periodic, aperiodic and random). Such a choice enabled an interpretation of the results for neuronal signals; another interpretation could be for trains of solitons. Yet, the model could be applied as well to investigate stochastic resonance in the propagation of arbitrary waveforms $s(t)$. Also, in our treatment the noise can be arbitrarily distributed, and the influence of the noise statistical distribution can be investigated. Even the consideration of a spatial arrangement of the noise varying along the line could be envisaged, although this has not been done here. The model is the first to demonstrate a noise-enhanced propagation based on an exact theoretical treatment applicable to periodic, aperiodic as well as random signals. It offers a unique framework for further studies on stochastic resonance and its potential applications.

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